

THERMAL CONVECTION IN A HORIZONTAL POROUS
LAYER WITH INTERNAL HEAT SOURCES

by

Morten Tveitereid

Department of Mechanics
University of Oslo

Abstract

Steady solutions in the form of hexagons and two-dimensional rolls are obtained for convection in a horizontal porous layer heated from within. The stability of the flows with respect to small disturbances is investigated. It is found that down-hexagons are stable for Rayleigh numbers R up to 8 times the critical value ($8R_c$), while up-hexagons are unstable for all values of R . Moreover, two-dimensional rolls are found to be stable in the range $3R_c < R < 7R_c$. Good agreement with some of the experimental observations of Buretta [1] is found.

Nomenclature

B_{pqh} ,	defined by (3.5);	∇ ,	$= (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$;
C_p ,	heat capacity at constant pressure;	∇^2 ,	$= \nabla \cdot \nabla$;
M ,	defined by (3.8);	∇_1^2 ,	$= \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$;
N ,	Nusselt number;	a ,	wave number;
Q ,	generated heat per unit time;	g ,	acceleration due to gravity;
R ,	Rayleigh number;	h ,	depth of the layer;
R_c ,	critical Rayleigh number;	k ,	permeability;
T ,	temperature;	p ,	pressure;
T_o ,	standard temperature;	\vec{v} ,	$= (u, v, w)$, velocity;
T_s ,	defined by (2.5);	t ,	time;
V ,	defined by (3.1);	x, y, z ,	cartesian coordinates.
ΔT ,	mean temperature differ- ence between the boundaries;		
ΔT_o ,	temperature difference between the boundaries for pure heat conduction;		

Greek letters

α	, coefficient of expansion;	μ	, viscosity;
$\vec{\delta}$, $= (\frac{\partial^2}{\partial x \partial z}, \frac{\partial^2}{\partial y \partial z}, -v_1^2)$;	ρ_0	, standard density;
θ	, temperature;	σ	, growth rate.
κ_m	, thermal diffusivity;		

Superscripts

\sim	, perturbation quantities;
$*$, complex conjugate quantities.

1. Introduction.

This paper is concerned with thermal convection in a porous medium. The convective motion is generated by internal heat sources which give a basic temperature gradient dT/dz varying with the vertical coordinate z . Porous convection is of considerable geophysical and technical interest, as it may occur in geothermal areas, through aquifers, oil reservoirs, snow layers, etc. (Combarnous & Bories [2].)

Porous convection when dT/dz being a constant, has been investigated by several authors. Horton & Rogers [3] and later Lapwood [4] determined analytically that above a certain dimensionless temperature gradient, convection can occur. Laboratory experiments have been performed by Schneider [5], Elder [6], Bories & Combarnous [7] and others. Theoretical and numerical analysis of finite amplitude convection have been performed among others by Elder [6], Palm, Weber & Kvernfold [8], Strauss [9] and Kvernfold [10].

In physical problems, however, a constant temperature gradient generally does not occur. Vertical variations of dT/dz may be due to variation in time of the temperature at the boundaries, or due to vertical variations of the thermal diffusivity for the porous medium. In the present analysis, however, the variation of dT/dz is thought of being due to uniformly distributed internal heat sources, which give a simple expression for the basic temperature.

To our knowledge, almost no research has been reported dealing with convection in a porous layer where dT/dz depends on z . Hwang [11] has studied the stability problem of convection in a porous layer with uniform heating from within and from below. He found that the critical Rayleigh number, R_c , decreases, as the effect of internal heating increases. For a model similar to the present model experimental studies have been performed by Buretta [1]. He measured the convective heat transport through the medium for Rayleigh numbers up to about $30R_c$. At a supercritical Rayleigh number, which appeared to depend on layer properties, a discontinuous jump in the convective heat transfer occurred. In view of this he postulated that R_c is a bifurcation point beyond which two finite amplitude modes of convection are possible.

In this paper we shall calculate the critical Rayleigh number.

Moreover, we shall derive steady solutions by a numerical technique, and examine the stability of these solutions with respect to small disturbances.

2. Governing equations

We consider a horizontally infinite layer of porous material saturated with fluid and heated from within by a uniform distribution of heat sources. The layer is bounded by two horizontal and impermeable planes separated by a distance h . The upper plane is taken to be perfect heat conductor and maintained at constant temperature, and the lower plane is taken to be perfect heat insulator.

In the Boussinesq approximation the equations governing the motion of the fluid may be written (Palm & Weber [12])

$$\nabla p + \rho_0 (1 - \alpha(T - T_0)) g \vec{k} + \frac{\mu}{k} \vec{v} = 0 \quad (2.1)$$

$$\nabla \cdot \vec{v} = 0 \quad (2.2)$$

$$\frac{(C_p)_m}{C_p} \frac{\partial T}{\partial t} + \vec{v} \cdot \nabla T = \kappa_m \nabla^2 T + \frac{Q}{C_p} \quad (2.3)$$

Here (2.1) is the equation of motion, (2.2) the continuity equation and (2.3) the heat equation. Moreover, p denotes the pressure, $v = (u, v, w)$ the velocity, T the temperature, ρ_0 a reference density, T_0 a reference temperature, α the coefficient of expansion, g the acceleration due to gravity, \vec{k} a unit vector directed upwards, ν the kinematic viscosity, k the permeability, κ the thermal diffusivity, C_p the heat capacity at constant pressure and Q the generated heat in the layer per unit time. The subscript m denotes properties of the fluid-solid mixture. We have chosen a

cartesian coordinate system (x, y, z) where the z -axis is directed upwards.

With the lower boundary at $z = 0$ the equations (2.1) - (2.3) are subjected to the boundary conditions

$$w = 0, \quad \frac{\partial T}{\partial z} = 0 \quad ; \quad z = 0 \quad (2.4)$$

$$w = 0, \quad T = 0 \quad ; \quad z = h$$

The temperature scale is chosen such that the constant temperature of the upper boundary is equal to zero.

When Q is small, the heat transfer is in the form of conduction ($\vec{v} = 0$). Let the static pressure and the conduction temperature then be denoted by p_s and T_s , respectively. From (2.3) and (2.4) we find that

$$T_s = \frac{Q}{2C_p \kappa_m} (h^2 - z^2) \quad (2.5)$$

For larger Q , in the convective regime, we write

$$\vec{v} = \vec{v}', \quad p = p_s + p', \quad T = T_s + \theta' \quad (2.6)$$

The equations may be written in a non-dimensional form by choosing h as a characteristic scale for length, κ_m/h for velocity, $(C_p)_m h^2 / C_p \kappa_m$ for time, $\mu \kappa_m / k$ for pressure, $\Delta T_o / R$ for θ' and ΔT_o for T_s . Here ΔT_o is the temperature difference between the planes for pure heat conduction and R the Rayleigh number, defined by

$$\Delta T_o = \frac{Q h^2}{2C_p \kappa_m}, \quad R = \frac{\rho_o k g \alpha \Delta T_o h}{\mu \kappa_m} \quad (2.7)$$

Omitting the primes the equations (2.1) - (2.4) then take the non-dimensional form

$$\nabla p - \theta \vec{k} + \vec{v} = 0 \quad (2.8)$$

$$\nabla \cdot \vec{v} = 0 \quad (2.9)$$

$$\nabla^2 \theta + 2RzW = \frac{\partial \theta}{\partial t} + \vec{v} \cdot \nabla \theta \quad (2.10)$$

with the boundary conditions

$$W = \frac{\partial \theta}{\partial z} = 0 \quad ; \quad z = 0 \quad (2.11)$$

$$W = \theta = 0 \quad ; \quad z = 1$$

The linearized version of (2.8) - (2.11) is an eigenvalue problem with $R = R_0$ as the eigenvalue. Introducing the wave number a , defined by

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = -a^2 \quad (2.12)$$

R_0 becomes a function of a . The minimum value R_c of R_0 for $a = a_c$ defines the value of the Rayleigh number for the onset of convection. $R_0 = R_0(a)$ is calculated by developing the solution in a power series of z . By applying 50 terms of this series, we found that

$$R_c = 30.933 \quad , \quad a_c = 2.448 \quad (2.13)$$

This result was checked by taking into account 100 terms without obtaining any changes of the given values.

3. Steady solutions

It follows from (2.8) and (2.9) that the velocity is poloidal, giving

$$\vec{v} = \vec{\delta}V = \left(\frac{\partial^2}{\partial x \partial z}, \frac{\partial^2}{\partial y \partial z}, -\nabla_1^2 \right) V \quad (3.1)$$

Thus, by eliminating the pressure term we obtain from (2.8) - (2.11)

$$\nabla^2 V = -\theta \quad (3.2)$$

$$\nabla^2 \theta - 2Rz \nabla_1^2 V = \vec{v} \cdot \nabla \theta \quad (3.3)$$

with the boundary conditions

$$V = \frac{\partial \theta}{\partial z} = 0 \quad ; \quad z = 0 \quad (3.4)$$

$$V = \theta = 0 \quad ; \quad z = 1$$

Considering solutions which are Fourier modes in the x, y -coordinates, θ may be written

$$\theta = \sum_{h=1}^{\infty} \sum_{p,q=-\infty}^{\infty} B_{pqh} e^{i(pkx+qly)} \cos(h-\frac{1}{2})\pi z \quad (3.5)$$

Here k and l are the components of the wave number vector in the x - and y -direction, respectively. Moreover, $B_{pqh} = B_{-p-qh}^*$ which ensure that (3.5) is real. The star denotes the complex conjugate. Corresponding to (3.5), V may be written

$$V = \sum B_{pqh} e^{i(pkx+qly)} F_h(\kappa, z) \quad (3.6)$$

where $\kappa^2 = (pk)^2 + (ql)^2$. $F_h(\kappa, z)$ is found from (3.2) and (3.4) (see appendix).

Introducing (3.5) and (3.6) into (3.3), multiplying this by $\exp\{-i(rkx+sly)\}\cos(g-\frac{1}{2})\pi z$ and averaging over the layer, we obtain a system of equations which determine the unknown coefficients B_{rsg} :

$$\begin{aligned} & \frac{1}{2}\{(g-\frac{1}{2})^2\pi^2 + v^2\}B_{rsg} - 2Rv^2 \sum_h a(h, v, g)B_{rsh} \\ & - \sum_{h,f} \sum_{\substack{p+t=r \\ q+u=s}} \{(ptk^2+qul^2)b(h, \kappa, f, g) + \kappa^2 c(h, \kappa, f, g)\}B_{pqh}B_{tuf} = 0 \end{aligned} \quad (3.7)$$

where $v^2 = (rk)^2 + (sl)^2$. The coefficients a, b and c are given in appendix.

The system of equations (3.7) has many different types of possible solutions. We shall, however, limit our analyse to the possibility of flow patterns consisting of hexagons or two-dimensional rolls. Probably, as in the case of free convection in a horizontal fluid layer, only hexagons and two-dimensional rolls can be stable flows for moderate Rayleigh numbers (Segel [13], Palm [14], Tveitereid & Palm [16]). To obtain this two types of flow as solutions of (3.7), we may require that all B_{rsg} are real, $B_{rsg} = B_{r-sg}$, $r+s$ equal to an even number, and that $k^2 = 3l^2 = 3a^2/4$. Moreover, we truncate the infinite system by neglecting all modes for which

$$g^2 + 3r^2/4 + s^2/4 > M^2 + 1 \quad (3.8)$$

Here M is an integer. In order to specify the values to be used for M , we introduce the Nusselt number N_M defined by

$$N_M = \Delta T_o / \Delta T = 1 / (1 + \sum_{g=1}^M B_{oog} / R) \quad (3.9)$$

where ΔT is the mean temperature difference between the planes and ΔT_o the temperature difference in the case of pure heat conduction. If N_M differs from N_{M-1} by less than 1%, the solution is accepted to be sufficiently accurate.

By using a Newton-Raphson method to solve (3.7), we find that two-dimensional rolls and both down-hexagons (i.e. descending flow in the centre of the cells) and up-hexagons are steady state configurations of our problem.

4. Stability analysis

Let $\tilde{\theta}$ and $\tilde{\vec{v}} = \tilde{\delta \vec{V}}$ denote a small variation of θ and \vec{v} , respectively. Furthermore, we assume an exponential time dependence

such that

$$\frac{\partial}{\partial t} \tilde{\theta} = \sigma \tilde{\theta} \quad (4.1)$$

where σ is the growth rate. By eliminating the pressure term, replacing \vec{v} with $\vec{v} + \vec{\tilde{v}}$ and θ with $\theta + \tilde{\theta}$ in (2.8) - (2.11), the equations governing the perturbations are

$$\nabla^2 \tilde{V} = -\tilde{\theta} \quad (4.2)$$

$$\nabla^2 \tilde{\theta} - 2Rz \nabla_1^2 \tilde{V} = \sigma \tilde{\theta} + \vec{\tilde{v}} \cdot \nabla \theta + \vec{v} \cdot \nabla \tilde{\theta} \quad (4.3)$$

with the boundary conditions

$$\begin{aligned} \tilde{V} = \frac{\partial \tilde{\theta}}{\partial z} &= 0 \quad ; \quad z = 0 \\ \tilde{V} = \tilde{\theta} &= 0 \quad ; \quad z = 1 \end{aligned} \quad (4.4)$$

Assuming periodical solutions in x and y , θ may be written

$$\tilde{\theta} = e^{i(\epsilon kx + \delta ly)} \sum_{pqh} \tilde{B}_{pqh} e^{i(pkx + qly)} \cos(h - \frac{1}{2})\pi z \quad (4.5)$$

where ϵ and δ are free parameters.

To obtain a complete stability analysis of the hexagonal flow δ and ϵ are varied from zero to one and from zero to $\delta/3$, respectively. For two-dimensional rolls with axis parallel the x -axis δ and ϵ are varied from zero to one and from zero to infinite, respectively.

From (4.2) and (4.5) we obtain

$$\tilde{V} = e^{i(\epsilon kx + \delta ly)} \sum_{pqh} \tilde{B}_{pqh} e^{i(pkx + qly)} F_h(\tilde{\kappa}, z) \quad (4.6)$$

where $\tilde{\kappa}^2 = (p + \epsilon)^2 k^2 + (q + \delta)^2 l^2$. We introduce (4.5) and (4.6) into (4.3), multiply with $\exp\{-i(\epsilon kx + \delta ly)\} \exp\{-i(rkx + sly)\} \cos(g - \frac{1}{2})\pi z$ and average over the layer. Then, an infinite set of linear and homogeneous equations determining B_{rsg} , follows. As in the previous section, we take into account only those equations for which $g^2 + 3/4r^2 + 1/4s^2 \leq M^2 + 1$. The Stability problem is thus reduced

to an eigenvalue problem with σ as the eigenvalue. If for given R and a at least one of the eigenvalues has positive real part, the examined flow is unstable.

5. Results and discussion

Figure 1 shows the results of the stability calculations. We find that down-hexagons and two-dimensional rolls are stable in a region of the (a,R) -plane, while up-hexagons are unstable for all values of a and R .

Hexagons.

The down-hexagons are stable in a rather small part of the wave number range from R_c up to $8R_c$. The stable region is tilted to the right, such that the wavelength of the cells at $8R_c$ is almost halved compared with the wavelength at R_c . Most of the curve which enclose the stable region (the neutral curve), is defined by non-oscillatory disturbances (i.e. $\sigma = 0$). From $4R_c$ up to $8R_c$, however, the left branch of the neutral curve is defined by oscillatory disturbances (i.e. the imaginary part of σ is different from zero). Moreover, also a subcritical region is found. This is, however, very small ($30.91 < R < R_c = 30.93$) and is of no practical interest.

The Nusselt number is illustrated in figure 2 for $M = 5$ and 6. We observe that N_6 differs from N_5 by less than 1%. This small difference, together with almost the same stable region for $M = 5$ and 6 (see figure 1), indicates that $M = 6$ defines an acceptable truncation of the equations.

The horizontally averaged temperature field, given by

$$\bar{T} = T_s + \frac{1}{R} \sum_{g=1} B_{00g} \cos(g-\frac{1}{2})\pi z \quad (5.1)$$

is shown in figure 3. As R is increased above R_c , we observe that the interior and the lower part of the layer become nearly isothermal, while a "thermal boundary layer" is formed in the upper part of the layer.

Rolls.

Two-dimensional rolls are stable in a broad wave number range from $3R_c$ up to $7R_c$. There is two types of disturbances which define the neutral curve. The right and the left branch are defined by cross roll disturbances, while the top branch is defined by Eckhaus disturbances (for a review of these types of disturbance: see Busse [15]). In the present case both the cross roll instability and the Eckhaus instability are non-oscillatory. To our knowledge the Eckhaus instability has never been observed in experiments. This is because the Eckhaus instability usually becomes important only for small supercritical Rayleigh numbers. The present result shows, however, that it should be possible to study the Eckhaus mechanism also in experiments.

The Nusselt number for steady two-dimensional rolls are given in figure 2 for $M = 5$ and 7 . We notice that N_7 differs from N_5 by less than 1%. Also the stable region is calculated for $M = 5$ and 7 . We find, however, the same neutral curve for this two values of M . Moreover, we observe the very small difference between the Nusselt numbers for rolls and hexagons. This fact supports the frequently used assumption that the convective heat transfer in a convection layer is nearly independent of the planform of the motion.

By comparing our results in the present work with the results

in [16, 17] we find agreement as to the sign of the circulation in the hexagonal cells. In that papers we found that down-hexagons are stable and up-hexagons unstable when the second derivative of the basic temperature is less than zero (as in the present case) and vice versa. The sign of the circulation is also in accordance with the observations in [18, 19].

For Rayleigh numbers from $3R_c$ up to $7R_c$ we found both stable rolls and stable hexagons. This bifurcation phenomenon must be due to properties of the porous medium. Since, in [16], where we studied convection in a fluid layer subjected to the same thermal conditions as the present ones, only down-hexagons were found to be stable.

Also different from the results in [16] is our finding of an upper limit of stable motion. However, the occurrence of unstable convection above $8R_c$ is probably caused by thermal instability of the thermal boundary layer. Let δ and R_δ denote the dimensionless thickness and the Rayleigh number of the boundary layer, respectively. Then

$$R_\delta = R\bar{T}(z=1-\delta)\delta \quad (5.2)$$

If $\bar{T}(z=1-\delta)\delta > 1/8$ at $R = 8R_c$, R_δ becomes larger than R_c . From figure 3 we observe that this condition may be fulfilled. In a fluid layer, however, where R_δ is proportional to δ^3 , R_δ does not become larger than R_c for moderate values of R .

Finally, also shown in figure 2 are the experimental values of N found by Buretta [1]. At a supercritical Rayleigh number R_D , depending on the diameter of the beads, he observed a remarkable discontinuity in the convective heat transfer. For R higher than R_D our numerical values of N agree very well with the experimental values. For R less than R_D , however, the agreement is rather bad. This discrepancy, we believe, is caused by experimental difficulties.

6. Summary

In this paper we have studied finite amplitude convection in a porous medium heated from within. Three different steady flows are analysed: down-hexagons, up-hexagons and two-dimensional rolls. By examining the stability of the flows with respect to small disturbances, the down-hexagons and the rolls are found to be stable planforms. The results of the stability analysis are shown in figure 1. Worth mentioning is also the occurrence of oscillatory instability of the down-hexagons. From $4R_c$ up to $8R_c$ the left branch of the neutral curve was defined by disturbances having a complex growth rate.

Moreover, in figure 2 we have compared the convective heat transfer with experimental values obtained by Buretta (1972). Some of the experimental values show good agreement with our values.

APPENDIX

Definition of the function $F_h(\kappa, z)$.

From (3.2) - (3.6) we obtain

$$\left(\frac{d^2}{dz^2} - \kappa^2\right)F_h(\kappa, z) = -\cos(h-\frac{1}{2})\pi z \quad (A.1)$$

with the boundary conditions

$$F_h = 0 \quad , \quad z = 0, 1 \quad (A.2)$$

We find

$$F_h(\kappa, z) = A_h(\kappa)\cos(h-\frac{1}{2})\pi z + C_h^{(1)}(\kappa)e^{-\kappa z} + C_h^{(2)}(\kappa)e^{\kappa z} \quad (A.3)$$

where

$$\begin{aligned} A_h(\kappa) &= 1/((h-\frac{1}{2})^2\pi^2 + \kappa^2) \\ C_h^{(1)}(\kappa) &= -A_h(\kappa)e^{\kappa}/(e^{\kappa} - e^{-\kappa}) \\ C_h^{(2)}(\kappa) &= A_h(\kappa)e^{-\kappa}/(e^{\kappa} - e^{-\kappa}) \end{aligned} \quad (A.4)$$

Definition of the coefficients a, b and c .

$$\begin{aligned} a(h, \nu, g) &= \int_0^1 z F_h(\nu, z) \cos(g-\frac{1}{2})\pi z \, dz \\ b(h, \kappa, f, g) &= \int_0^1 F_h'(\kappa, z) \cos(f-\frac{1}{2})\pi z \cos(g-\frac{1}{2})\pi z \, dz \\ c(h, \kappa, f, g) &= (f-\frac{1}{2})\pi \int_0^1 F_h(\kappa, z) \sin(f-\frac{1}{2})\pi z \cos(g-\frac{1}{2})\pi z \, dz \end{aligned} \quad (A.5)$$

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FIGURE LEGENDS

Figure 1. The stable regions.

——— , the neutral curve for down-hexagones.
Stable inside, unstable outside.

..... , the neutral curve for two-dimensional rolls.
Stable inside, unstable outside.

..... , the marginal stable curve.

Figure 2. The Nusselt number as a function of the Rayleigh number.

o, ●, values of the Nusselt number from Buretta [1].

Figure 3. The horizontally averaged temperature for the hexagonal flow.

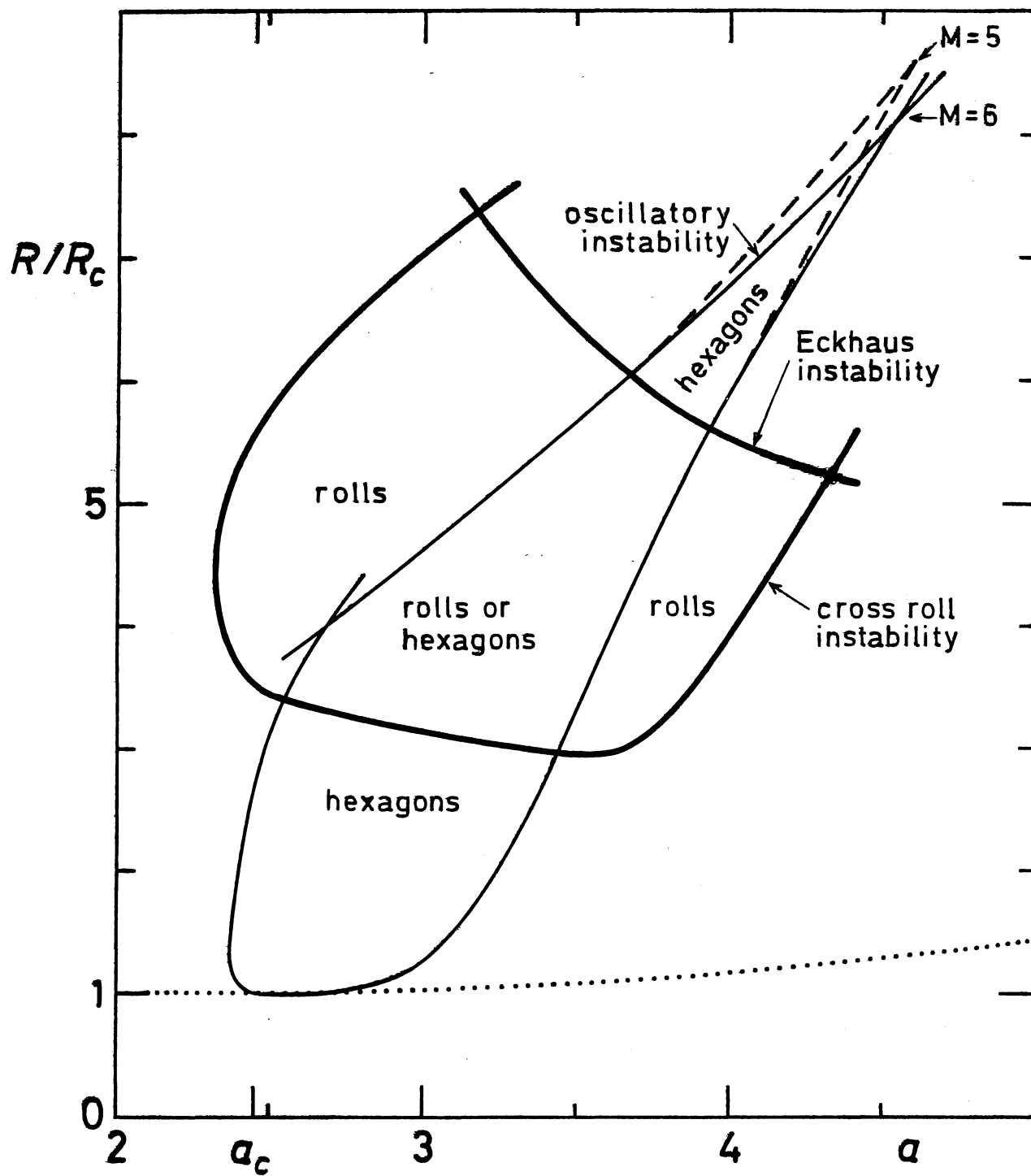


Figure 1

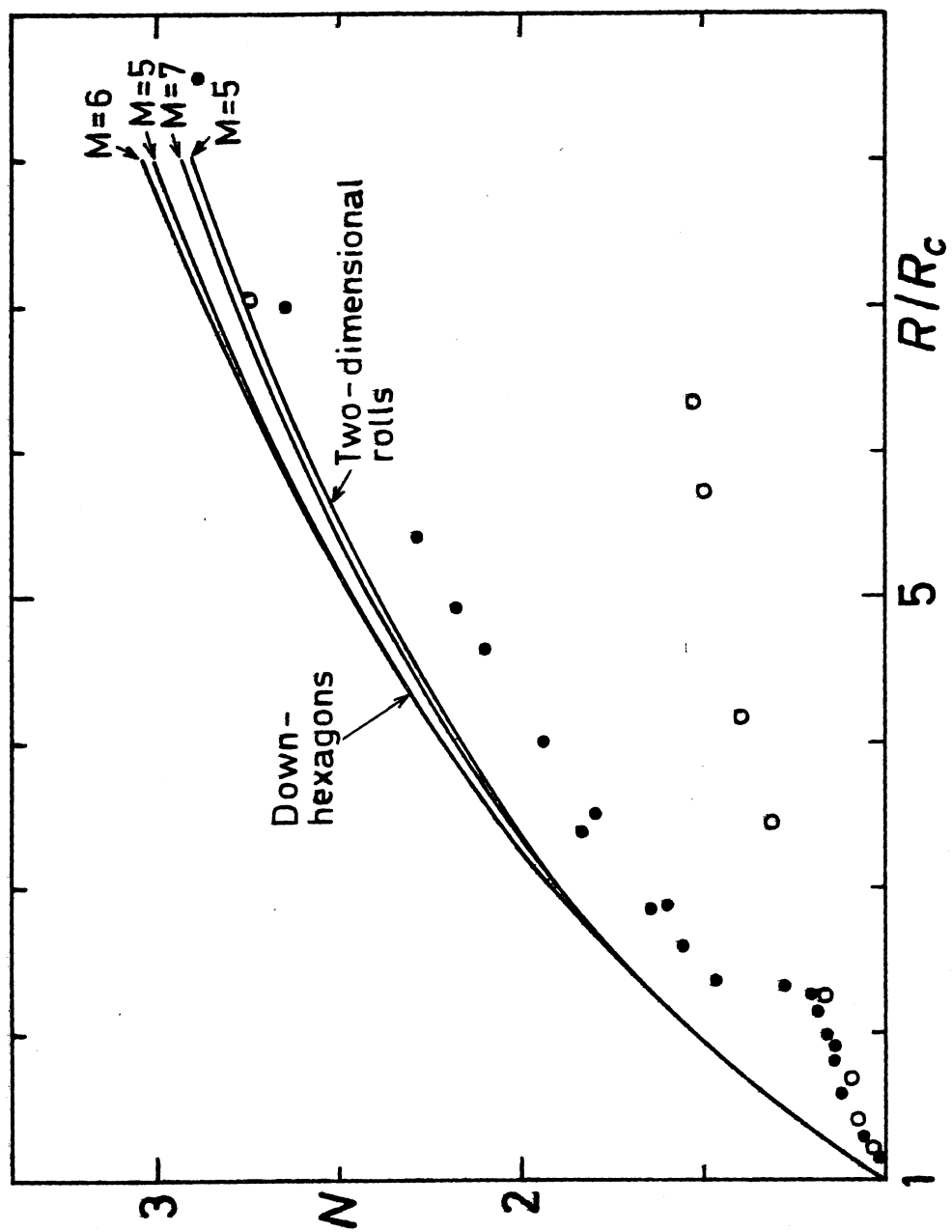


Figure 2

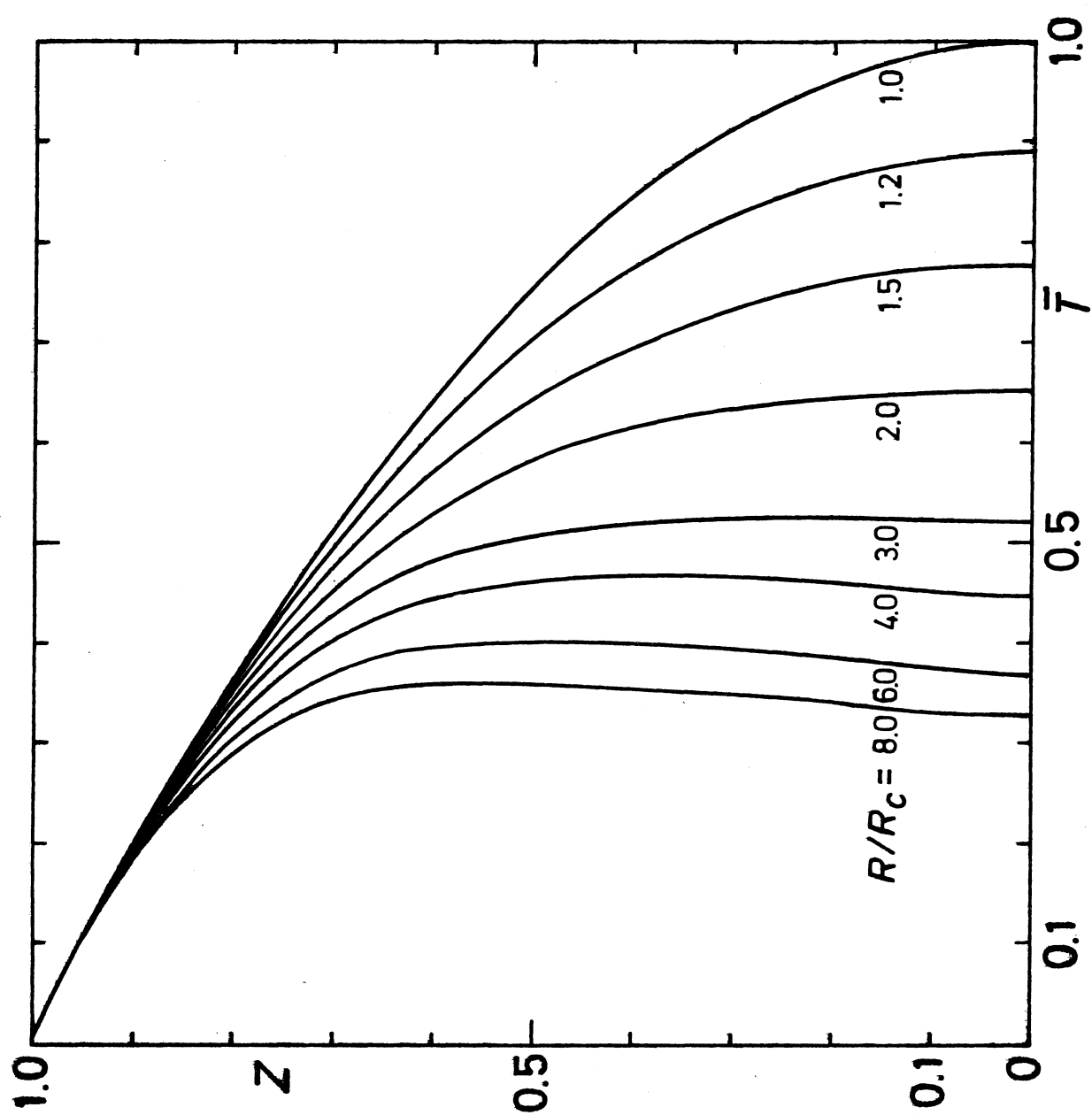


Figure 3